Week 1

In(tro)duction

Thorben Klabunde

22.09.2025

Agenda

1. Meet & Greet
2. About the Course
3. Theory Recap
4. Assignment
5. Tips & Tricks

Meet & Greet

About Me

- **Name:** Thorben Klabunde
- **Hometown:** Winterthur, CH
- **Hobbies:** Swimming & cooking

About You

- Now it's your turn so we all get to know each other!
- Please briefly introduce yourself:
  - **Name**
  - **Hometown**
  - **Hobbies**

About the Course

Organizational

- **Schedule & Contact**
  - **Schedule**: LEE C104, Mo. 09:00-12:00
  - **Contact**: tklabunde@student.ethz.ch
  - **Resources & materials**: www.th-kl.ch

Organizational

- **Exercises & Bonus Points**

  During the semester, you can get bonus points for

  - solving the designated parts of the **theoretical exercise sheets** (in working groups);
    3 points per exercise sheet;
  - solving the **multiple choice quizzes** in the beginning of the exercise class (individually);
    1 point per week starting in week 3;
  - **peer grading** the specified part of the theory sheets (in working groups);
    1 point per exercise sheet;
  - solving the **programming problems** (individually);
    6 points per exercise sheet.

  You can find all details regarding grading on the Moodle Course Website. Please make sure to read it carefully!

Organizational

- **Exercises & Bonus Points**

  BUT don't stress out too much!

  - You only need **80% of the points** (!) and even if you have no bonus points, you can still get the highest mark.
  - **Code-Expert** is worth a lot of points, which you have plenty of time for.
  - **Quizzes** give you valuable feedback but are **only worth 1 point** (I certainly didn't have full marks on quite a few quizzes, no need to worry!)
  - **Assignments do not need to be as formal as the master solution**. I will try to guide you and give you feedback.

Organizational

- **Feedback-System:**

  Although it might seem like a lot, the system is designed to help you!

  - **Working-Groups:**

    Partners for assignment submissions (newly assigned by me every 3 weeks).

    Solutions to exercise sheets are submitted in pairs (only one submits solution on Moodle).

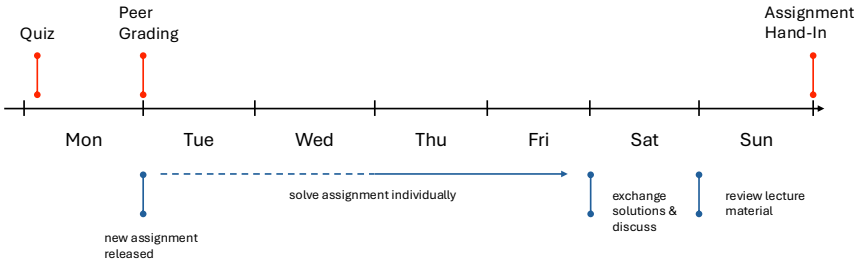
    Use the chance to get to know some of your peers!

  - **Peer-Grading:**

    Forces you to take an outside perspective and notice what assumptions to make explicit.

Weekly Schedule for A&D

This ended up being my schedule for A&D last fall just to give you a sense. At the start of the term, you'll have a lot more time and will finish assignments much sooner. Don't worry when it shifts back slightly!



A&D Overview

Weeks

1-5

Basics & Complexity

- Asympt. Notation
- Induction
- Loop Counting

Search

- Linear Search
- Binary Search
- Lower Bound

Sorting

- Bubble Sort
- Selection Sort
- Insertion Sort
- Merge Sort
- Quick Sort
- Heap Sort
- Lower Bound

5-7

Data Structures

ADTs

- List, Stack, Queue, Dictionary

Data Struct.

- Arrays, Lists, Heaps, Trees

Dynamic Programming

- Fibonacci Numbers
- Max. Subarray Sum
- Jump Game
- Longest Common Subseq.
- Edit Dist.
- Subset Sum
- Knapsack
- Longest Ascending Subseq.

8-14

Graph Theory

Introduction

- Definitions & Properties
- Topological Sort

Graph Search

- Depth First Search – DFS
- Breadth First Search – BFS

Shortest Path

Single-source SP

- BFS
- Djikstra
- Bellman-Ford

All-pair SP

- Floyd-Warshall
- Johnson
- Matrix Mult.

Minimum Spanning Trees

- Prim
- Boruvka
- Kruskal

Exam

- **4h split btw. theory and coding** - the exam format has been revised this term, I will update you once I know more
- The **assignments ideally prepare you for the exam**! Try to do all of them!

Questions?

Website: www.th-kl.ch **Feedback:** Google Forms (anonymous)

Theory Recap

Karatsuba's Algorithm: Idea & Correctness

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  2. Factoring out yields:

     $$xy = \mathbf{ac} \cdot 10^{2m} + (\mathbf{ad} + \mathbf{bc}) \cdot 10^m + \mathbf{bd}$$

     But this still needs **4 multiplications**

Karatsuba's Algorithm: Idea & Correctness

- **Key Idea:** Notice that

$$(a+b)(c+d) = ac + ad + bc + bd \tag{1}$$
$$= ac + (ad + bc) + bd \tag{2}$$
$$\overset{\text{arithm. rearr.}}{\Longleftrightarrow} \quad (\mathbf{ad} + \mathbf{bc}) = (a+b)(c+d) - \mathbf{ac} - \mathbf{bd} \tag{3}$$

Only **3 multiplications** required!

Pasture Break

Pasture Break

Setting: A shortsighted cow is at an 'origin' point on a very long, circular fence of length l.

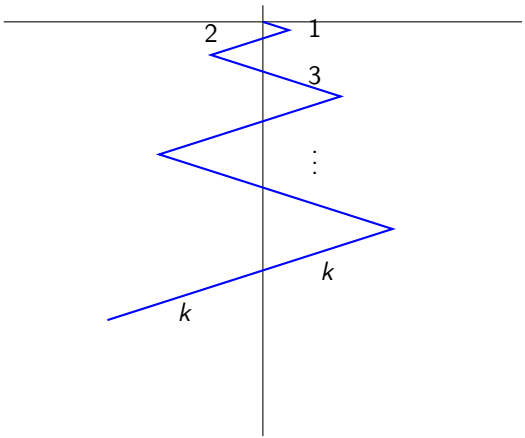
Goal: Find the closest gap in the fence.

Constraints: The closest gap is k steps away from the origin, but k is unknown. The total fence length l is much larger than k ($l \gg k$). The cow can only see the gap when standing directly next to it.

Pasture Break: Linear Step-Sizing

Strategy I: Linear Step-Sizing

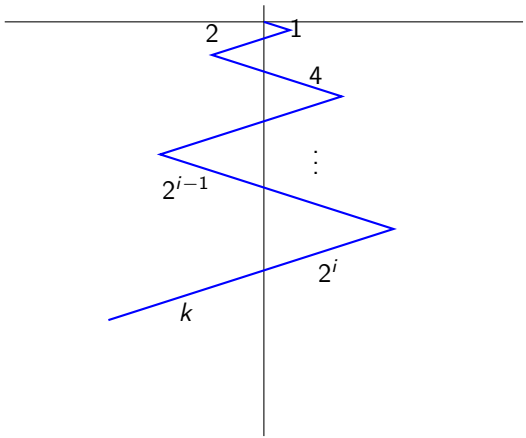
- Proceed in a zig-zag pattern, increasing the length one step at a time.
- **Number of Steps (worst case):**



Pasture Break: Exponential Step-Sizing

Strategy II: Exponential Increase in Step-Size

- Proceed in a zig-zag pattern, doubling the search distance after a failed attempt.
- **Number of Steps (worst case):**



Proof by Induction - baby steps go a long way!

Recall: When do we use a proof by induction? How do we proceed?

Proof by Induction - baby steps go a long way!

Proof By Induction

Used to prove statements of the form $\forall n\ P(n)$, where the universe is the set $\{k, k+1, k+2, \ldots\}$ for some $k \in \mathbb{N}$.

Consists of **two steps**:

Proof by Induction - baby steps go a long way!

Proof By Induction

Used to prove statements of the form $\forall n\ P(n)$, where the universe is the set $\{k, k+1, k+2, \ldots\}$ for some $k \in \mathbb{N}$.

Consists of **two steps**:

1. **Basis step:** Prove $P(k)$.
2. **Induction step:** Prove that for arbitrary $n \geq k$, $P(n) \implies P(n+1)$.

The induction step is performed by assuming $P(n)$ (the **induction hypothesis**) and deriving $P(n+1)$.

Proof by Induction - A Strategic Workflow

1. **Formulate the claim**: What are we trying to prove?
2. **Identify the Induction Variable**: Determine the variable, n, and its starting value, k.
3. **Identify the Base Case(s) (B.C.)**
   Think ahead! The structure of the Induction Step (I.S.) determines how many base cases you must prove.
4. **State the Induction Hypothesis (I.H.)**
   - *Weak Induction*: Assume $P(n)$ is true for an arbitrary integer $n \geq k$.
   - *Strong Induction*: Assume $P(i)$ is true for all integers i such that $k \leq i \leq n$.
5. **Prove the Base Case(s)**
   Beware, sometimes there are multiple.
6. **Prove the I.S.** under the assumption of the I.H.

Assignment

Exercise 0.1.a)

Exercise 0.1 *Induction.*

(a) Prove by mathematical induction that for any positive integer n,

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$

In your solution, you should address the base case, the induction hypothesis and the induction step.

Exercise 0.1.a) - continued

Exercise 0.1.b)

(b) **(This subtask is from August 2019 exam).** Let $T : \mathbb{N} \to \mathbb{R}$ be a function that satisfies the following two conditions:

$$T(n) \geq 4 \cdot T(\tfrac{n}{2}) + 3n \qquad \text{whenever } n \text{ is divisible by 2;}$$
$$T(1) = 4.$$

Prove by mathematical induction that

$$T(n) \geq 6n^2 - 2n$$

holds whenever n is a power of 2, i.e., $n = 2^k$ with $k \in \mathbb{N}_0$. In your solution, you should address the base case, the induction hypothesis and the induction step.

Exercise 0.1.b) - continued

Exercise 0.2.b)

(b) $f(n) := n^3$ grows asymptotically faster than $g(n) := 10n^2 + 100n + 1000$.

Exercise 0.2.c)

(c) $f(n) := 3^n$ grows asymptotically faster than $g(n) := 2^n$.

Exercise 0.3.a)

(a) $f(n) := n^{1.01}$ grows asymptotically faster than $g(n) := n \ln n$.

Exercise 0.3.b)

(b) $f(n) := e^n$ grows asymptotically faster than $g(n) := n$.

Exercise 0.3.d)

(d)* $f(n) := 1.01^n$ grows asymptotically faster than $g(n) := n^{100}$.

Exercise 0.3.f)

(f) $f(n) := 2^{\sqrt{\log_2 n}}$ grows asymptotically faster than $g(n) := \log_2^{100} n$.

Exercise 0.4

Exercise 0.4 *Simplifying expressions.*

Simplify the following expressions as much as possible without changing their asymptotic growth rates.

Concretely, for each expression $f(n)$ in the following list, find an expression $g(n)$ that is as simple as possible and that satisfies $\lim_{n\to\infty} \frac{f(n)}{g(n)} \in \mathbb{R}^+$.

Exercise 0.4.d)

(d) $f(n) := 23n + 4n \log_5 n^6 + 78\sqrt{n} - 9$

Exercise 0.4.e)

(e) $f(n) := \log_2 \sqrt{n^5} + \sqrt{\log_2 n^5}$

Exercise 0.4.f)

(f)* $f(n) := 2n^3 + (\sqrt[4]{n})^{\log_5 \log_6 n} + (\sqrt[7]{n})^{\log_8 \log_9 n}$

Tips & Tricks

Workflow

Disclaimer: This is my workflow and is based on my experience and what works for me. I hope it helps you discover a suitable workflow for yourself. Don't let anyone's opinions sway you too much if you've found something that works.

Workflow - Typed Note-Taking

Note-Taking Style

- Active-Recall using Question-Answer style notes is great for revision
- Would recommend a structured note-taking app (see below) and advise against flash-card apps like Anki

  Used Anki in first term and found that it lacked structure for look-ups and revision

Note-Taking App:

Notion lets you do both - structured note-taking & active-recall style notes (with toggled bullets)

- **Pros:** Easy, clean, well-formatted note-taking with nice shortcuts and features (inline latex, code-blocks with syntax highlighting, toggle bullets, search)
- **Cost: Free** - Sign-up with your student mail to get a free Plus Plan.

Workflow - Handwritten Notes

GoodNotes

- **Pros:** Clean and well-structured user-interface, nice pens & templates.
- **Cost: 10CHF/year (iOS)** - Not free but reasonable pricing.